Measurement-based and Blind quantum computation

Berlin School of
Optical Sciences &
Quantum Technologies

Outline

Measurement based quantum computation

- Instead of *gates*, use *measurements* to drive the computation

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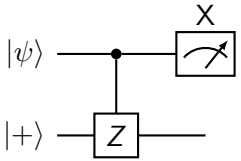
- Only single-qubit measurements needed
- Uses a *resource* - not just qubits, but (highly) entangled ones
- Equally powerful as gate-based computation

Basics - Teleportation



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$$|\psi\rangle_t = Z^m H |\psi\rangle$$

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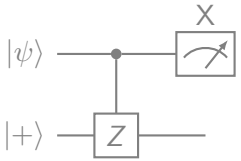

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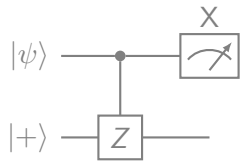

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$M(\theta) := \{|0\rangle \pm e^{i\theta} |1\rangle\} = R_z(\theta) \cdot X$

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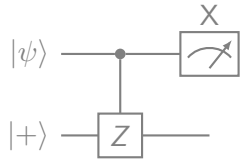

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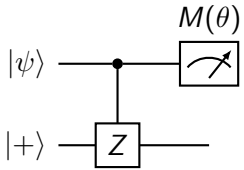

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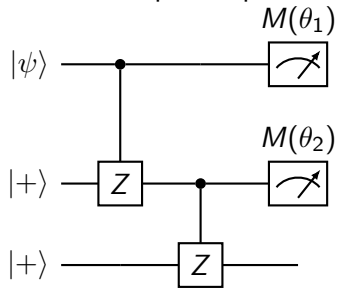

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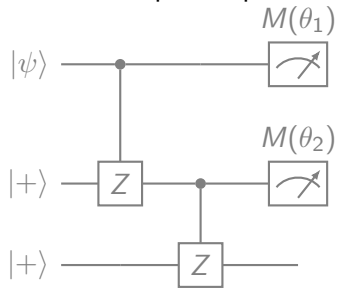

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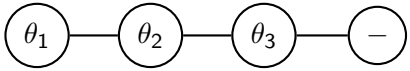
Three times gives us:

$$HR_z(\theta_2)\underbrace{HR_z(\theta_2)H}_{R_x(\theta_2)}R_z(\theta_1)\,|\psi\rangle$$
$$\sim U(\theta_1,\theta_2,\theta_3)\,|\psi\rangle$$

A different way of representing

The cluster state:



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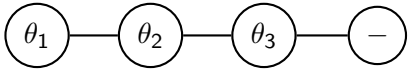

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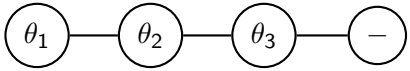

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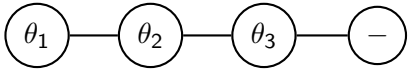

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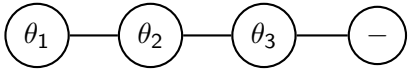

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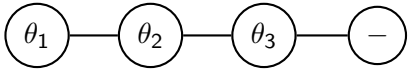

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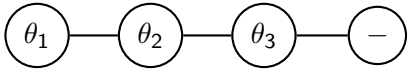

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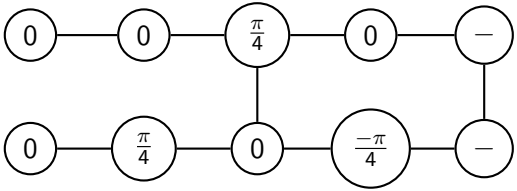
- This input 'resource' is the same for all measurement patterns

What about 2-qubit gates?



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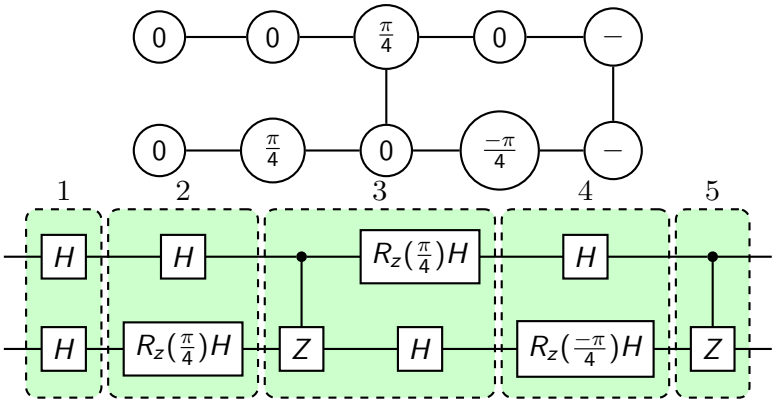

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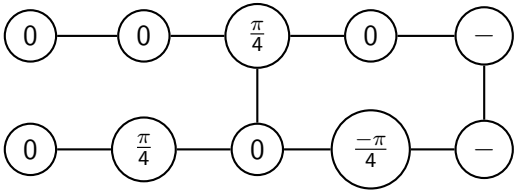

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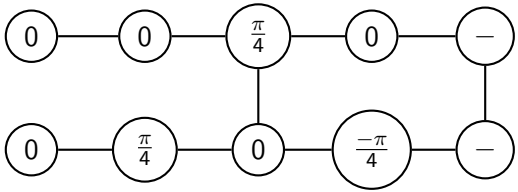
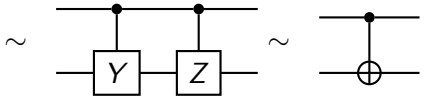
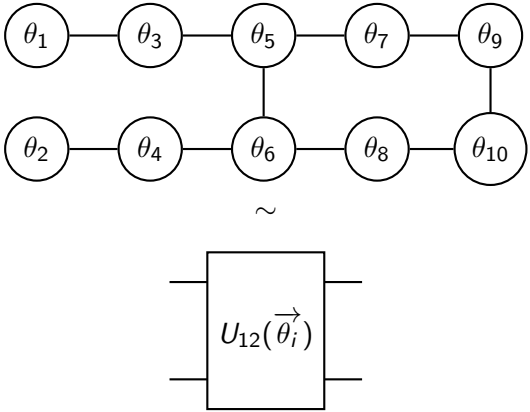




A *Universal* resource: the *brickwork* state



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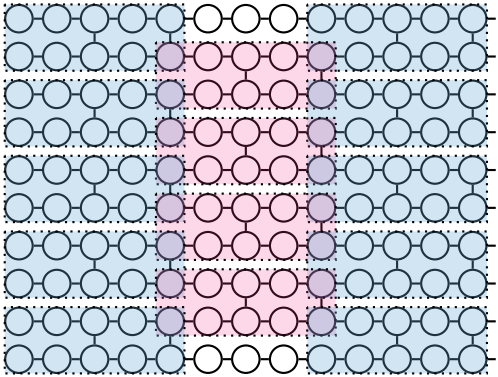

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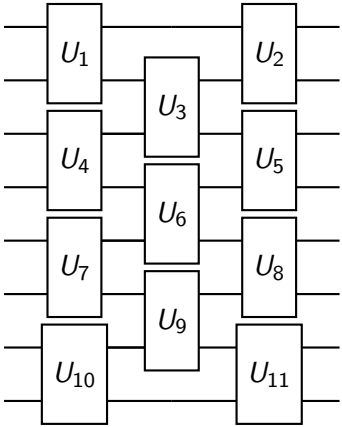

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- n-qubit circuit with gate depth of g gives n rows and $O(g)$ columns
- $BQP \subset$ MBQC (and obviously vice-versa)

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- Still $O(ng)$ scaling

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- MBQC and gate-based equally powerful

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- Verifiable (no tricks from the server!)

Multiple protocols

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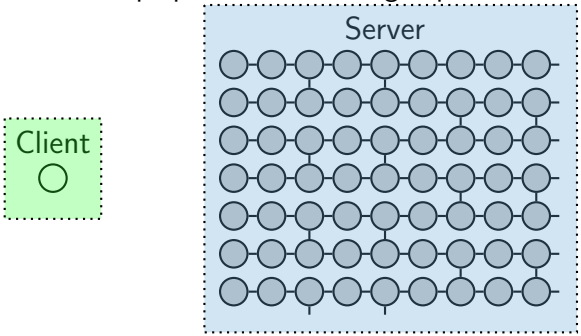
First protocol[1]

Client can prepare and send single qubits



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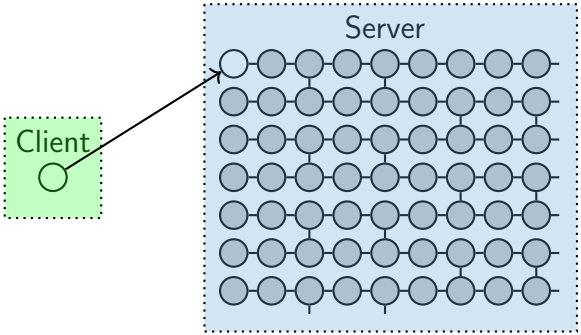

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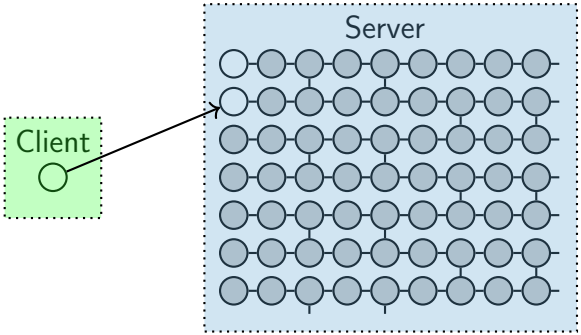

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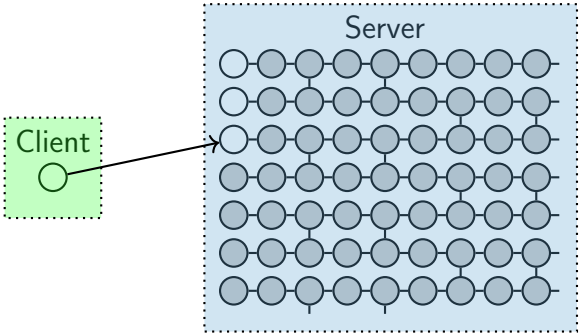

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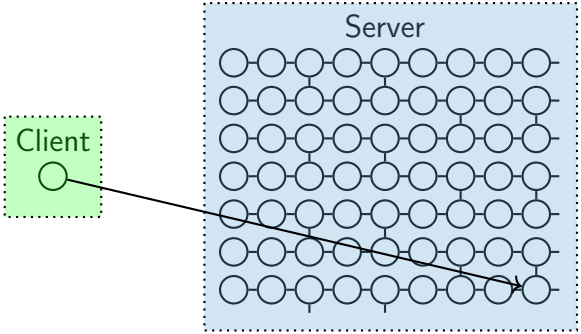

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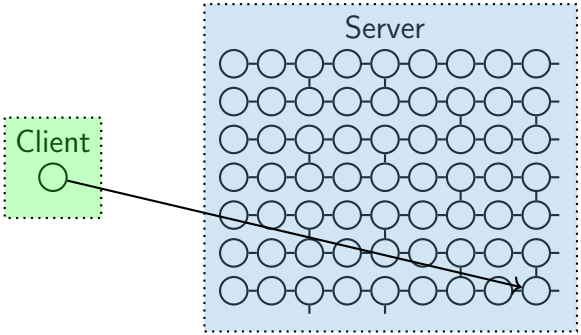

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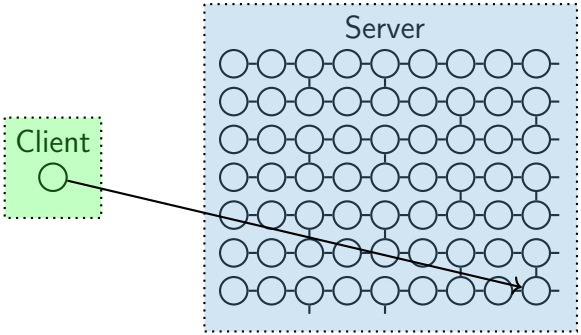

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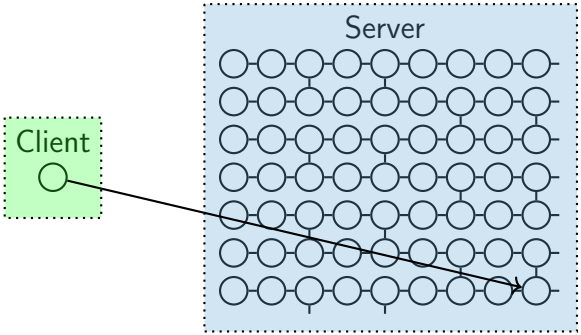

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Hide the measurement angles

Protocol

For every node i, with intended measurement angle θ_i:

- Client sends qubit $|\phi_i\rangle = |0\rangle + e^{i\phi_i}|1\rangle = R_z(\phi_i)|+\rangle$
- Client picks random $r_i \in \{0, 1\}$
- Client tells server to measure with angle $\hat{\theta}_i = \theta_i + \phi_i + r_i\pi$

Measuring $|\phi_i\rangle$ with angle $\hat{\theta}_i \hat{=}$ measuring $|+\rangle$ with angle $\theta_i + r_i\pi$

- Server sends measurement outcome $m_i \in \{0, 1\}$
- Client computes output $o_i = m_i$ or $o_i = 1 - m_i$

Security

- Server wants:

  Classic information: n, g, $\{\theta_i\}$, $\{o_i\}$

- Server receives:

  Classic information: n, g, $\{\hat{\theta}_i\}$, $\{m_i\}$
  Quantum information: $\{|\phi_i\rangle\} = |0\rangle + e^{i\phi_i}|1\rangle$

  - Classic leakage:

    $\hat{\theta}_i$, θ_i are uncorrelated due to random ϕ_i
    $\{m_i\}$, $\{o_i\}$ are uncorrelated due to random r_i
    No leakage but n, g

  - Quantum leakage:

    $\{|\phi_i\rangle\} = |0\rangle + (-1)^{r_i} e^{i(\hat{\theta}_i - \theta_i)}|1\rangle$
    Trace away r_i: maximally mixed state
    No leakage whatsoever

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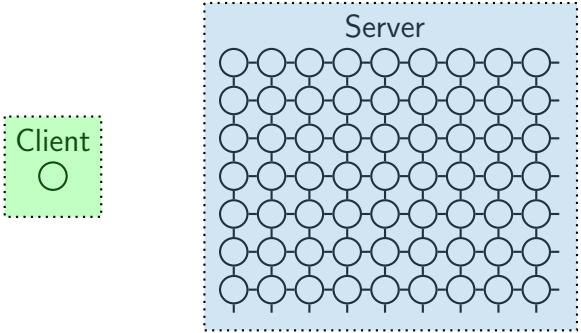

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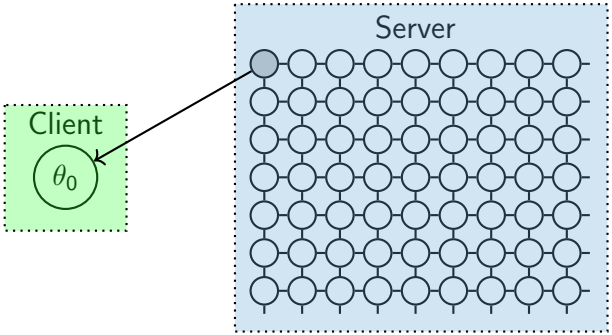

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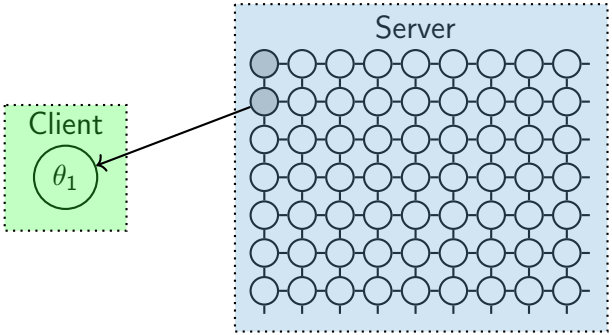

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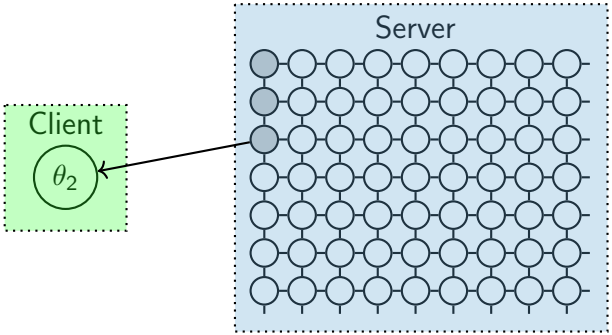

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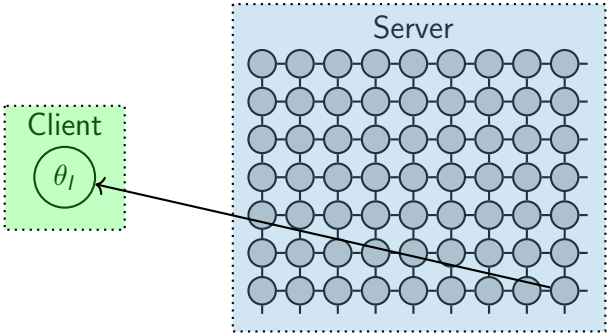

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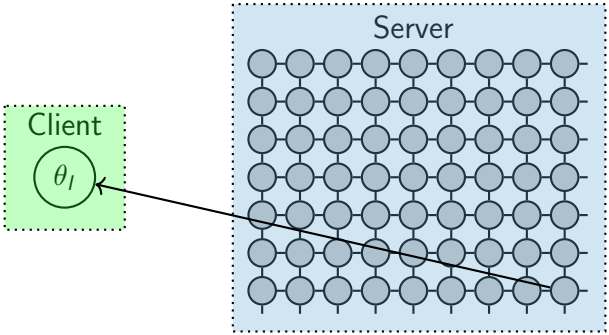

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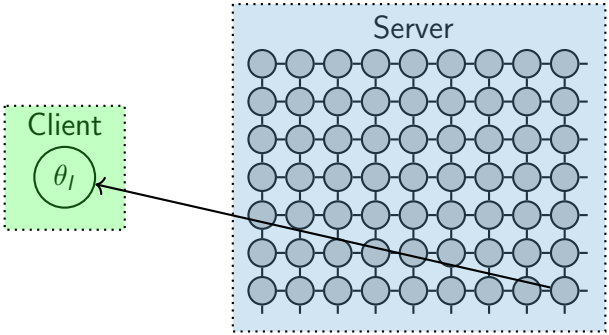

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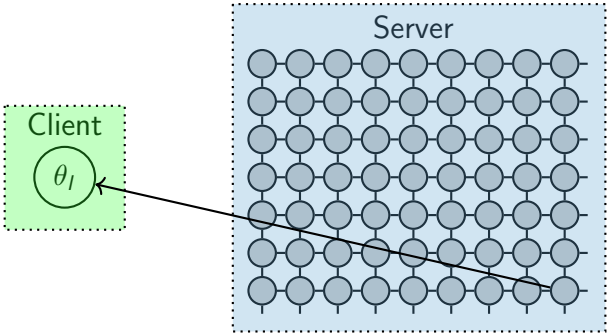

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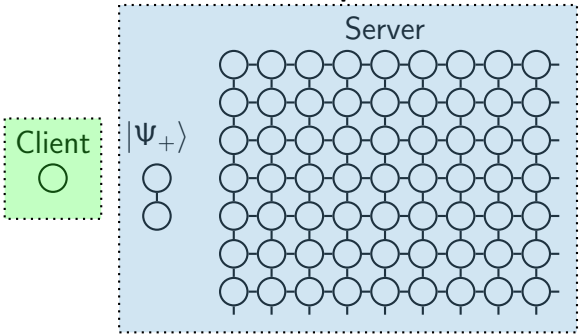
Very prone to photon loss

Second protocol #1.*b*

Client can receive, measure single qubits
and store them for short periods in time



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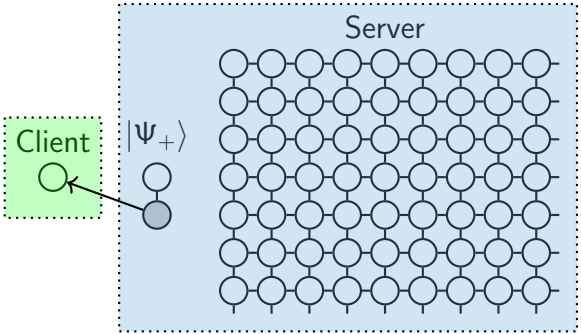

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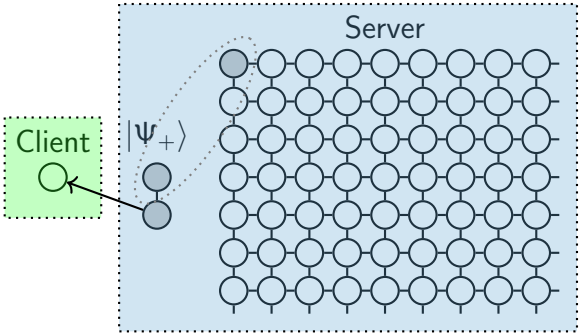

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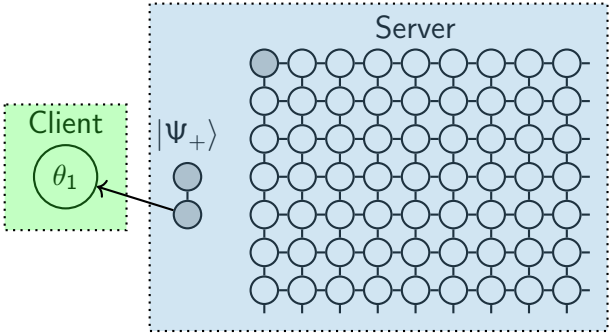

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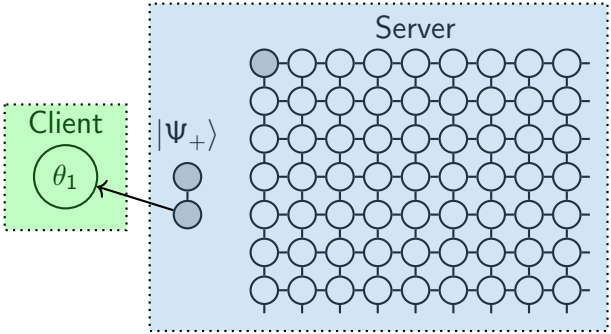

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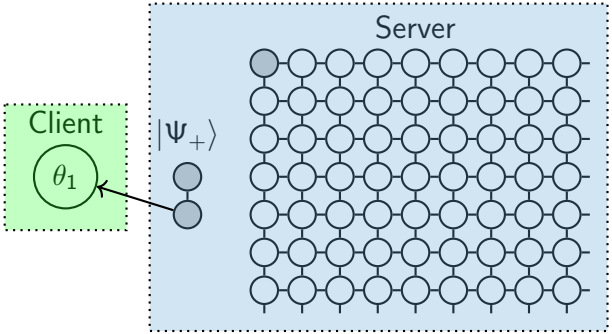

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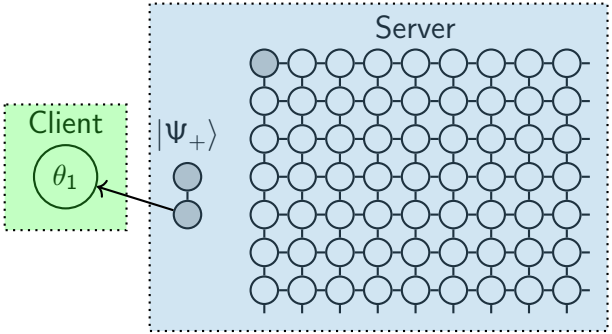

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Qubit storage not a nice aspect...

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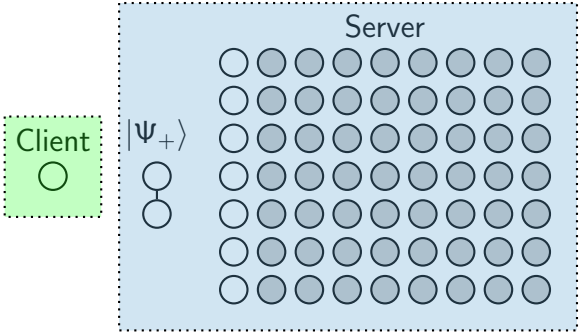
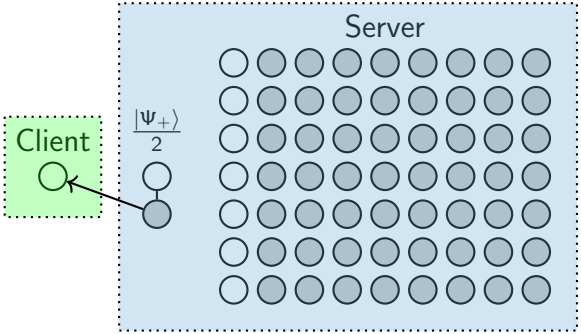


Create a Bell pair

Second protocol #2



Client receives half

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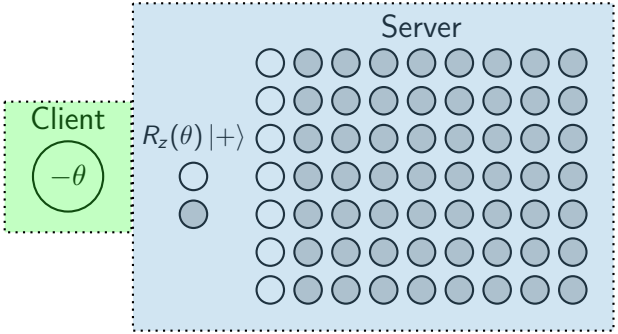
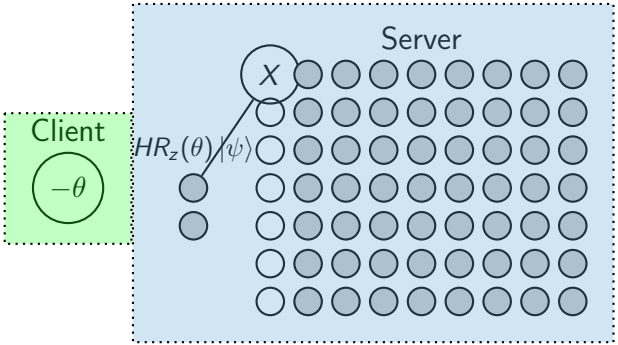


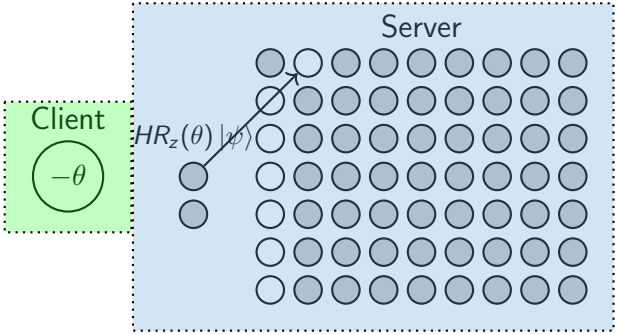
Client measures in $-\theta$ basis, other half becomes $R_z(\theta)\,|+\rangle$

Second protocol #2



Entangle with node $|\psi\rangle$, measure node in X, get $HR_z(\theta)|\psi\rangle$

Second protocol #2



Send $HR_z(\theta)\,|\psi\rangle$ to its 'right' place in the chain

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Version #2 is secure by the same argument

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- Other questions about i.e. verifiability

Thanks for the attention!

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Multiple teleportations with -1 outcomes

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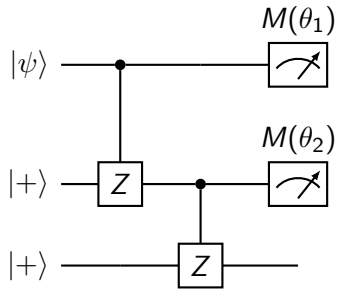

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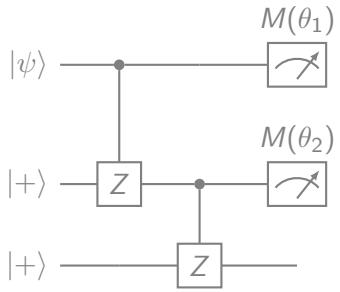

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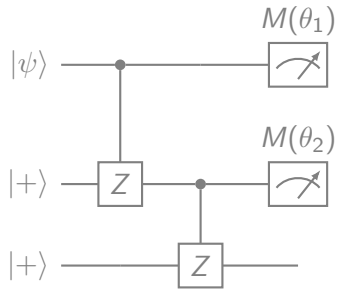

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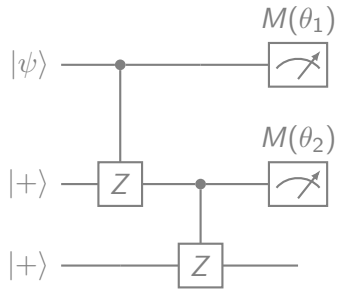

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Paulis can be pulled through, but **measurement angles become dependend of previous outcomes**

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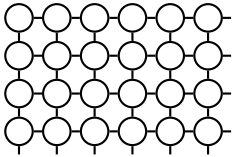
- Measure stabilizers of the resource every now and then

Backup - Different kinds of resources

- We saw the *brickwork* state



- Originally introduced was the *cluster* state
- With θ- and Z-measurements
- Only in 2017 it was shown[1] that one doesn't need Z-measurements for universality
- Many other resources possible

[1] Mantri et al. - Sci Rep 7, 42861 (2017)

Backup - A note on the two-qubit unitary